A TECHNIQUE FOR THE DEVELOPMENT OF AN INDEX OF RELATIVE TAXPAYING ABILITY OF LOCAL ADMINISTRATIVE UNITS

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CHAPTER I

INTRODUCTION

Development of State Support

The problem of adequately financing public education by equitable methods is an old problem which has been given increasing attention in recent years. Variations in assessment of property, variations in financial ability, variations in the concept of the school program, and variations in willingness to pay for education have complicated this problem. Although the shift from local to state support has solved some of the problems, it has created others. One problem created is the development of techniques for apportioning state aid to local administrative units.

One of the earlier studies by Cubberley[1] in 1906 focused attention on the large number of children who were deprived of a decent educational opportunity. This structural deficiency resulted from a lack of local financial ability, as well as a general weakness of organization. Despite the fact that the property tax was burdensome, an adequate yield for public education was not forthcoming in many local school units. Several developments of the twentieth century have accentuated this disparity of opportunity.

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[1] E. P. Cubberley, School Funds and Their Apportionment.

The per cent of school age children attending school has grown from 58.6 per cent[2] in 1900 to 71.4 per cent[3] in 1947-48. This growing demand for education on the part of the public contributed to the increase in state support. The limited program undertaken by the nineteenth century school has expanded to a curriculum broad enough to strive for the full development of the child. Such enrichment and diversification of offerings brought with it a longer school year, expansion of plant, more highly trained instructional personnel, and hence greater unit costs. Increased industrialization in urban areas caused further concentration of wealth in these same areas. In an era when the government has taken greater responsibility for the security and welfare of the people, the right to a decent educational opportunity has become more widely recognized.

State support of education has been in operation since the early days of our government. Connecticut in 1795 undertook the entire cost of the school program from state resources. New York inaugurated state aid in 1812 on a modified basis; New Jersey followed in like manner in 1815.[4] Although the methods of distribution were naturally crude, this is evidence that even then some financial responsibility was being felt by educational leaders within a few

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[2] Twelth Census of the United States: 1900. Population: Number and Distribution of Inhabitants, p. xcv.

[3] The Forty Eight State School Systems, p. 182.

[4] Paul R. Mort, State Support of Education, p. 90.

states. In general, however, the plan of educational finance throughout the first part of the nineteenth century was one of local support.

As long as school costs were small, the need of a comprehensive plan of financing education was not generally recognized. With the emergence of the industrial revolution, educational inequalities both in opportunities and in tax burdens became more pronounced. In an effort to cope with this breakdown of local support, attempts were made to equalize the burden through state aid. The Massachusetts Law of 1874 provided for aid only to the rural schools. In 1881 New Jersey passed a law giving the State Board of Education authority to distribute funds at its own discretion.[5] By 1900 equalization laws were operating in eight states, primarily eastern states.

The idea of state support was well established by 1890, when the average participation of all the states was 23.8 per cent. The majority of the southern states contributed more than 50 per cent of the educational costs that year and continued this proportion for about two decades. The over-all average gradually declined to 16.0 per cent in 1925. By 1931 the average rose to 17.3 per cent but no state paid for as much as half its educational program.[6] Since 1930 the trend toward greater state support has been significant. In 1937-38 ten states contributed more than half the revenue, while the

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[5] Ibid., p. 33.

[6] Ibid., p. 26.

average was 29.6 per cent. In 1947-48 fifteen states supplied more than half the revenue, while the average was 39.8 per cent.[7]

Principles for the Distribution of State Aid

By 1920 the need for clarifying the issues of state support became so strong that the American Council on Education sponsored from 1921 to 1924 an extensive survey of the problem. This was called the Educational Finance Inquiry.[8] Most state support investigations in the past twenty five years have been based on the statement of principles enunciated by this survey:

> To carry into effect the principle of 'equalization of educational opportunity' and 'equalization of school support' as commonly understood, it would be necessary (1) to furnish the children in every locality within the state with equal educational opportunities up to some prescribed minimum; (2) to raise the funds necessary for this purpose by local or state taxation adjusted in such a manner as to bear upon the people in all localities at the same rate in relation to their taxpaying ability; and (3) to provide adequately either for the supervision and control of all the schools, or for their direct administration by a state department of education.

This definition combines two well entrenched principles fundamental to our American democracy, "equal opportunity for all" and "equality of the burden of taxation." Translated into its simplest terms, it means taxing property where it is, and educating the children where they are. The prescribed minimum in (1) above

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[7] The Forty Eight State School Systems, p. 224.

[8] George D. Strayer and Robert M. Haig, Financing of Education in the State of New York, p. 174.

has become generally known as the minimum foundation program. Acceptance of this definition demands that the burden of financing a minimum foundation program falls equally among all the people and areas of the state according to their taxpaying ability. Many states have had an equalization program without a minimum foundation program; as long as money is distributed so that poorer districts receive more state money per pupil than wealthier districts, such a plan may be said to follow the equalization principle. In 1947-48 forty three states[9] employed some type of equalization in distributing educational funds.

The idea of the minimum foundation program was a natural outgrowth of Strayer and Haig's concept of equalization and has been accepted by many states as a major objective of school finance. Mort[10] defined the defensible minimum foundation program for each state as the cost of the program provided in a district of average wealth. This level may vary considerably from state to state; it does not assume to be what the expenditures per child ought to be; but it is a good first step, and is practical from the standpoint of likely acceptance by the people and the state officials.

Another definition of the minimum foundation program was formulated by Johns in the Report of the State Education Commission of North Carolina:[11]

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[9] The Forty Eight State School Systems, p. 236.

[10] Paul R. Mort, The Measurement of Educational Need, p. 8.

[11] Education in North Carolina Today and Tomorrow, pp. 90-91.

> The minimum foundation program of education is comprised of the 'educational decencies' which every child in the American democracy has a right to expect. Stated simply, the foundation program is a well educated, inspirational teacher competent to bring out of a child the best that is in him; a well lighted, well heated, sanitary, attractive school building well equipped with good books and other instructional materials; safe, hygienic transportation if he lives too far from school to walk; and, finally an efficient plan for operating and maintaining the school plant and administering the school system. It is made up of those basic essentials of education which should be made equal for every child in the state regardless of his race or where he lives.

In recent years the trend has been more toward the definition given in the North Carolina survey. The level of the minimum foundation program in both Florida[12] and Texas[13] was considerably above the average maintained in those states.

Various stages of development are found in state foundation programs. The level of foundation support in most states is considerably below that necessary to assure a modern educational program that the people would accept as a satisfactory minimum. The minimum cost per unit, then, must be considered a dynamic element in an evolutionary period. It should be recognized that the expenditure level is transitory and may be expected to rise as the people become more aware of its desirability.[14]

The variance of need, ability, and organizational background

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[12] Florida School Law, 1947. An Act Pertaining to Education.

[13] Texas School Law, 1949. Foundation School Program Act.

[14] Guides--to the Development of State School Finance Programs, p. 14-15.

of the various states has produced several different approaches to financing a foundation program. Some states have provided the entire support of the minimum foundation program, such as North Carolina and Delaware. This method is called the large fund plan. The necessity of measuring the relative taxpaying ability of the local units is eliminated in this plan. The most popular method is called the small fund plan, whereby local administrative units form a partnership with the state in financing the program. Although some controls and standards have been found necessary at the state level, the local administrative unit thrives best when vested with a maximum amount of initiative and freedom in administering the schools.[15] Then how shall the state distribute its share to the local administrative units?

From the time of Cubberley's pioneer study[16] in 1905 to the release of The Educational Finance Inquiry Commission in 1921, the principles underlying state support had been classified roughly as equalization and efficiency. The interpretation of the efficiency principle, also known as "payment-for-effort," was substantially changed by the analysis of Strayer and Haig.[17] They showed that the equilization principle tended to shift the tax burden to the abler communities, but that the payment for effort principle tended to

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[15] Ibid., p. 6.

[16] Cubberley, op. cit.

[17] Strayer and Haig, op. cit.

shift the burden to the less able communities. Only financially capable districts were able to participate. Subsequent analyses based on more recent state fiscal programs indicate less attention being given to payment for effort. Mort's interpretation of the equalization principle includes the factor of minimum effort if the burden of a satisfactory minimum educational program is to be shared equitably. So in the evolution of principles for the distribution of state aid, the principle of payment-for-effort has shifted from the direct approach, or matching technique, to the more equitable uniform minimum effort in relation to taxpaying ability.

The equalization plan of distributing state aid is based on two component parts: need, and effort in relation to ability. The first technical attack on determining educational need was contributed by Mort[18] in 1924, and made possible a practical application of the equalization principle. His research was directed toward the determination of cost units of educational need which would allow for variance in class size of schools operating under different circumstances. Mort also advocated that each local unit be required to assume its share of the burden of financing the minimum foundation program according to its relative financial ability. This local requirement may be defined as effort in relation to ability.

Any state attempting to support a partnership finance program must seek for as accurate and equitable an estimate of taxpaying

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[18]Paul R. Mort, The Measurement of Educational Need.

ability of each local administrative unit as possible. Four methods of estimating taxpaying ability are being used:[19]

(1) Local assessments. This method is highly unsatisfactory because it varies considerably from county to county with respect to true valuation. Secondly, it is subject to local manipulation.

(2) State supervised assessments. This is some improvement over (1) in that the opinions of impartial officials are brought to bear. However, local officials still share in the valuation of property.

(3) State Tax Commission. Such bodies appraise the true value of property in each local unit. This method should be the most desirable, since a direct measure of what we are trying to measure is obtained without bias or manipulation.

(4) Index of taxpaying ability. In this method an objective technique is sought that will predict relative ability on the basis of the economic factors of wealth contained in the local administrative units.

As of 1947-48, twenty four states based local effort on local assessments, eight states used a State Tax Commission or state supervised assessments to estimate actual values, and with the recent addition of Texas, seven states now employ some type of an index of taxpaying ability.[20]

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[19] Guides to the Development of State School Finance Programs, pp. 17-18.

[20] The Forty Eight State School Systems, p. 236.

Statement of the Problem

The index of taxpaying ability is the one method of measurement that can be thoroughly objective in the absence of accurate appraisals. The importance of the economic index has been promoted in recent years, because of the reluctance of local administrative units to give up the power of property assessments. Only a few states have succeeded in establishing some form of state tax commission with the power of assessment. Until this success becomes more widespread, the economic index of ability offers the best substitute. However, a standard technique for producing a workable formula for predicting ability has not been fully perfected. It is the purpose of this study to develop a non-empirical technique or a formula that will more accurately predict relative ability. Standards for the development of the formula will be set forth; an evaluation will be made of some historical and technical contributions pertinent to this problem; the characteristics of the desired function will be described; the selection of factors and criteria will be discussed; on the basis of the given standards and requirements a technique will be proposed and evaluated.

CHAPTER II

REVIEW OF PERTINENT DEVELOPMENTS

Standards for the Development of an Index of Ability

In the second study of the series published by the National Survey of School Finance,[1] two bases for the distribution of federal aid to the states were discussed--educational need, and income produced. As Associate Director of this Survey, Mort formulated criteria for the development of indices for either of these bases of distributing aid. These standards were:

1. All measures used must be objective.
   This is to avoid friction between the states and the central agency arising from ambiguity of measures; to avoid local manipulations of data to increase the amount of aid received; and to avoid the tendency to bureaucratic control that tends to arise from any plan which is not sufficiently objective to avoid the need on the part of the central agency for exercising judgment.

2. Measures used should be based to the greatest degree possible upon dependable data systematically obtained by federal agencies at determined periods and by tried and established methods.
   This is to avoid the setting up of cumbersome additional machinery; to avoid local manipulation of data; to free the federal government from delay due to inefficient record systems in the state; and to insure uniformity in the collection of information.

3. Measures used must be as equitable as they can be made without the introduction of complexity.

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[1] Mort, Research Problems in School Finance, pp. 133-37.

4. Federal aid must be non-fluctuating in nature.

The federal aid should be sufficiently stable to enable careful planning by states. It should be possible for any state to predict the amount of aid it shall receive sufficiently far ahead to make state legislative action possible. For example, this requires that the amount of aid which a given state gets shall be determined by data arising from that state alone. It should not depend upon the developments in all of the other states.

5. The plan must be such as to have a common-sense appeal both in the measures applied and in the scheme of distribution.

Experience has shown that measures obtained by refined methods are not necessarily lacking in their appeal as common-sense measures.

6. The plan must not in any way interfere with the rights of the states to shape their own educational destinies.

It should be stated that the problem of determining the tax-paying ability of states is not the same problem as determining the ability of localities within a state. However, the techniques used in developing an index and the standards governing the measures to be used are quite similar. The following standards were adapted by Cornell[2] from the National Survey of School Finance in developing measures of the ability of the counties of New York:

1. All measures used should be objective.

This is to avoid friction between the local units and the state arising from ambiguity of measures; to avoid manipulation of data to increase the amount of aid received; and to avoid the tendency to bureaucratic control that tends to arise from any plan which is not sufficiently objective to avoid the need on the part of the central agency for exercising judgment.

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[2]Francis G. Cornell, A Measure of Taxpaying Ability, pp. 17-19.

2. Measures used should be based to the greatest degree possible upon dependable data systematically obtained by governmental agencies at regular intervals and by tried and established methods.

This is to avoid the setting up of cumbersome additional machinery; to avoid local manipulation of data; to free the state from delay due to inefficient record systems in the localities; and to insure uniformity in the collection of information.

3. Measures used should be as equitable as they can be made without the introduction of undue complexity.

4. State aid should be non-fluctuating in nature. - The state aid should be sufficiently stable to enable careful planning by localities. It should be possible for any local unit to predict the amount of aid it shall receive far enough ahead to make possible proper budgeting of local expenditures. For example, this requires that the amount of aid which a given locality gets shall be determined by data arising from that locality alone. It should not depend upon the developments in all the other localities of the state.

5. The plan should be such as to have a common-sense appeal both in the measures applied and in the scheme of distribution.

Experience has shown that measures obtained by refined methods are not necessarily lacking in their appeal as common-sense measures.

6. Statistical suitability. Mathematical techniques must be resorted to in combining measures and only through numerical treatment of data is it possible to judge the adequacy of techniques in terms of the other requirements.

These standards were applied not only to the selection of individual measures, but also to the index combining these measures.

As Special Finance Consultant for the South Carolina Survey[3] of 1948, Johns proposed standards for the development of an index of the

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[3] Public Schools of South Carolina. A Report of the South Carolina Educational Survey Committee, p. 322.

relative taxpaying ability of the counties of South Carolina. Some of these were used by Mort and Cornell, others were added by Johns. These standards were:

1. An index of the relative taxpaying ability of the several counties of South Carolina to support schools must not be an index of theoretical taxpaying ability but a measure of relative wealth in order to be equitable because local taxes for schools are largely derived from property.

2. A sufficient number of economic measures of wealth should be included in the index to include all the principle elements of the wealth of the state on which taxpaying ability is based.

3. All economic measures should be objective and accurate and obtainable from reliable published sources.

4. All economic measures included in the index should be independent from the influence of local assessing bodies.

5. Each economic measure should measure some different aspect of the wealth of the state insofar as possible.

6. Statistically, each economic measure should have a fairly high positive correlation with the true wealth of the respective counties of the state but the economic measures should have as low intercorrelation with each other as possible.

7. The mathematical formula employed for the development of the index of taxpaying ability should measure the relative taxpaying ability of small counties as accurately as large counties.

The guiding standards used in this present study are adapted from criteria used by Johns. These standards are:

1. The index and all economic factors should be objective; therefore, all data pertaining to these factors should be obtainable from reliable published sources.

2. All economic factors and the index should be independent

of the influence of local assessing bodies.

3. Each economic factor should measure some different aspect of the wealth of the state, and a sufficient number should be included in order to represent all the principal elements of the wealth of the state.

4. The index should be based on some validating measure which directly corresponds to the actual value of property.

5. The mathematical formula employed for the development of the index of taxpaying ability should be as sensitive to the small local units as it is to the large local units in predicting relative ability.

6. The index of taxpaying ability should be as equitable as possible without undue complexity in order that the formula be administratively feasible.

Method of Least Squares

It might seem that the linear multiple regression equation[4] by the method of least squares would satisfy the requirements of these standards. It offers a precise method of fitting a curve describing the relationship among measurable variables. A number of workers have attempted to utilize this technique in attacking the particular problem of this study. That the sum of the squares of

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[4]Hereafter when the expression multiple regression equation is used, it should be understood that it is a linear multiple regression equation by the method of least squares.

the residuals be a minimum is the test of goodness of fit in a great many problems where a predictive formula is desired. The least squares technique is the answer to such problems by its very derivation because it guarantees the sum of the squares of the deviations (residuals) to be a minimum. However, there is a fundamental reason why the unmodified least squares method does not justify application to a formula of relative taxpaying ability of local administrative units. In our problem the magnitude of the errors of measurement is influenced by the magnitude of the quantity being measured. The standards set up for the development of a formula require that equitableness for the small local units be as important as it is for the wealthy local units.

As an illustration, suppose the criterion of metropolitan county A is given to be 15 per cent of the state's total property valuation, and the criterion of rural county B is given to be 0.5 per cent. According to the least squares technique, a prediction of 14 per cent for county A and 1.5 per cent for county B would show an equal error of 1 per cent. However, the relative error for county A would be a nominal 6.7 per cent, as compared to the relative error for county B of 200 per cent. As is shown later in this study empirical weighing methods have been devised which actually will yield a smaller sum of the relative residuals than that obtained by the method of least squares, and which therefore are more equitable according to the given standards.

There is a second characteristic of a regression equation by the method of least squares that prevents direct application to this problem. It is well known by statisticians that a regression formula determined by least squares may yield a computed prediction greater or less than any of the independent variables in the equation. For example, let us consider the counties of the state of Tennessee: the criterion is the full value of property; and the economic factors selected for the state are auto registrations, sales tax returns, gainfully employed workers, and farm income. Each variable is expressed in terms of its per cent of the state total (see Table 2 in Chapter VI). If X_1, X_2, X_3, X_4 represent these four factors respectively, the multiple regression formula by the method of least squares for the counties of Tennessee is calculated to be

$$X_p = .100X_1 + .287X_2 + .504X_3 + .200X_4 - .097.$$

For the sake of simplicity the values of the economic factors are expressed in per cents throughout this study. For Pickett County these values are respectively, .075, .036, .062, and .194. Substitution of these values in the formula gives Pickett County an index of -.009 per cent of the state's taxpaying ability. This negative index of course has no meaning. The example of Pickett County not only illustrates that the least squares formula can predict outside the range of the evidence given by the economic factors, but that it can also give a meaningless index. The exaggerated importance of the constant with respect to the smaller local

units, and the not unlikely occurrence of negative coefficients permit such predictions. Any technique that allows such predictions is neither equitable nor administratively feasible. Therefore, the direct application of a multiple regression equation by the method of least squares to this statistical problem of relative taxpaying ability must be rejected. Mort, Cornell, and Johns were all cognizant of the shortcomings of the method of least squares and it will be seen later in this chapter how they sought to overcome these obstacles.

Strayer and Haig

One of the first attempts to go beyond assessed valuation as a measure of relative ability was done by Strayer and Haig.[5] With the purpose of improving on the measure of local ability, the New York State Tax Commission made a complete study of the taxable income of each county in 1921. In addition, the full value of taxable real estate was determined by this Board, so that two measures instead of one could be used to predict ability. After numerous trials on the proper weights of these two factors, this index of economic resources was formulated: One half the sum of (a) taxable income, plus (b) one tenth full value of real estate. This meant that a dollar of income was given ten times the importance of a dollar of real estate. Trials using one fifth, one fifteenth, and one twentieth of the real

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[5]Strayer and Haig, op. cit., pp. 167-73.

estate value revealed no significant changes in the ranks of the counties of New York. Despite the crudeness of this technique, it represents a significant contribution as one of the first steps toward combining real estate values with a measure of income to determine relative taxpaying ability.

Mort

Mort in his published research[6] on state support gives major credit to the work of Strayer and Haig on the Educational Finance Inquiry Commission in developing the equalization principle of state support. Recent trends of state support indicate that the general pattern developed by Mort has gained wide acceptance. In 1949 forty three states were using some type of equalization method in distributing state education funds.[7] The plan of state support developed by Mort involves the following steps:

1. Determination by objective methods of the cost of the minimum foundation program to be guaranteed by the state to all children.

2. Determination of the proportion of the total cost of the foundation program to be paid from state funds and the proportion from local taxes.

3. Determination of minimum tax effort in relation to taxpaying

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[6] Mort, State Support for Public Education.

[7] The Forty Eight State School Systems, p. 236.

ability which must be made by each local unit toward support of the minimum foundation program in that unit.

4. Calculation of the state equalization funds to which each local unit is entitled by finding the difference between the cost of the minimum foundation program for each local unit and the amount of funds which it can raise locally by making the state required minimum tax effort as related to its taxpaying ability.[8]

Mort was fully cognizant of the necessity for using an equitable measure of local taxpaying ability in connection with the apportionment of state equalization funds. He pointed out that the full evaluation of property is the best measure of local ability, because property taxes represent the sole source of taxation which most local boards are permitted to assess.[9] The problem, however, as has been already pointed out, is to determine what is the full value of property in each local unit.

One of the important differences between an index of ability for states and an index of ability for local units arises from the tax sources available to the states as contrasted to the local units. The state and nation, it would seem, have unlimited sources of taxation for all their respective expenditures. Hence, an index applying to the states may properly be an index of the theoretical

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[8] Mort, State Support for Public Education.

[9] Ibid., pp. 16-17.

taxpaying ability, whereas an index applying to local administrative units should be an index of taxpaying ability according to those sources of taxation available to local school units. Mort used the computed yield of a "uniform modern tax system" (composed of about six types of taxes) as the criterion for validating his index of taxpaying ability for the states of the Union and the District of Columbia. The development of the index embodied three steps:

1. The selection of factors
2. The weighting of factors
3. The validation of the final index.

No attempt has been made to evaluate the validity of the ten factors used by Mort, since the purpose of this study is centered on the evaluation of weighting techniques. Because many of these factors were used in subsequent studies, they are set forth below:

1. Urban population
2. Value added by manufacture
3. Farm cash income weighted inversely as the percentage of farm population of total population
4. Postal receipts
5. Retail trade -- net sales
6. Motor vehicle registrations
7. Value of net incomes reported for federal income tax returns
8. Value of net incomes reported for federal income tax returns. ($25,000 and over)
9. Total population
10. The yield of a theoretical tax on one-half of one per cent of the par value, or, in the case of no par stock, of the issue price on authorized capital stock of corporations[10]

Mort expressed all data in units per capita for reasons of expediency where the requirement of simplicity was essential for

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[10]Mort, Federal Support for Public Education, pp. 192-93.

possible federal aid legislation. Calculations were made in such a way that the results were the equivalent of, first, expressing all variables as percentages of the United States total, and second, weighting each in inverse ratio to the population reported for the state.[11] A multiple regression formula for the forty eight states and the District of Columbia of the per-capita yield of selected taxes as the criterion was determined on nine of the above factors. This formula was converted to a form for finding a state's actual tax yield in terms of the factors expressed in the aggregate. Then the formula was reconverted so that the per cent distribution on the yield of the selected taxes could be predicted from the factors when they were expressed as per cents of the total. In the process of these two conversions the original constant became the coefficient of the factor population. Finally, factor (10) was introduced by means of several adjustments. These coefficients adjusted to total unity were then taken as the weights.[12]

The fundamental objection to Mort's technique is his use of the method of least squares to determine the coefficients of the factors in the index. Two measures of the relative influence of the factors--Beta coefficients and part correlation coefficients[13]--were computed by Mort but were not employed. The use of as many as ten economic

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[11] Loc cit.

[12] Ibid., pp. 311-12.

[13] Ibid., p. 314.

factors is unnecessarily cumbersome, and may be reduced without significant loss of accuracy. In expressing all data in units per capita, Mort sought to overcome the limitations of the least squares method and incorporate some idea of relative residuals into the development of the formula. This procedure was justified (1) on the grounds that the exaggerated weighting of the more able and populous states would be minimized, and (2) as an expedient to gain simplicity. However, such a procedure does not seem to be applicable at the state support level. Cornell[14] states:

> Unfortunately, when the unit under consideration is smaller than the state, greater heterogeneity with respect to factors influencing ability tends to affect per capita measures from the statistical standpoint. For a number of useful measures the per capita relationship with ability is practically zero.

Although Mort never intended to use this index to predict local ability, his technique is examined in the light of its possible application. Numerous adjustments and conversions were required for the final determination of Mort's index. This characteristic impairs the adaptability of his technique for calculating the taxpaying ability of local school units. A completely objective technique of combining measures would enable independent experts to achieve identical results.

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[14] Cornell, op. cit., p. 37.

Newcomer

An estimate of the relative ability of the states[15] was developed by Newcomer on the basis of the yield of a model tax system. This measure was adopted by Mort as the criterion for the development of an index of the relative ability of the states.[16] Newcomer based her study on the hypothesis that the theoretical ability of the states would be the best criterion for use in a federal support program. Since the states were fiscally sovereign, an ideal tax system theoretically could be set up. The use of the entire tax system as a measure of ability was rejected for two reasons: (1) the index was unduly complex, and (2) the influence of benefit taxes would be out of proportion to their importance. The yield of the following selected six taxes was suggested as the most practical index of ability: personal income, real estate, severance, business income, corporation organization, and stock transfer. Newcomer discussed four difficulties in the use of the yield of a model tax system: tests of ability were imperfect; the index was influenced by chance conditions; the statistical data were inadequate; ability was a variable factor. The comprehensive investigations of Newcomer provided much-needed research in the field of theoretical ability, but her general technique would not apply to local school administrative units.

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[15] Mort, Federal Support for Public Education, pp. 117-178.

[16] Ibid., p. 193.

Cornell

In 1936 Cornell[17] published a study offering two techniques of measuring taxpaying ability of the counties of New York State. Assessed valuation, full value of real property,[18] and aggregate taxes on property were all used as criteria in numerous experiments. According to both methods of developing a formula, it was found that the weightings applied to the six economic factors employed by Cornell yielded indices of relative ability which agreed with full valuation better than would assessed valuation. He showed that these indices could be developed by using either full value of property or aggregate taxes on property as a criterion. Assessed valuation was rejected as a criterion. Experiments leading to formulas in both logarithmic form and linear form were advanced.

Use of logarithms was suggested when the graphed relationships between the criterion and the individual measures tended to be more linear on log scaled graphs than on the equally spaced arithmetic graphs. Using the method of least squares and expressing all the variables in terms of logarithms, a formula was developed based on 51 of the 62 counties of the state. The five counties of Greater New York and six other counties were treated separately, because they were found to have special characteristics not common to the other

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[17]Cornell, op. cit.

[18]Equalization rates determined by the New York State Tax Commission were applied to county assessed valuations.

counties. Empirical corrections increasing the index of relative ability were made to these counties which had one or more of these characteristics: large population increases, sparsity of population, small gross production per inhabitant, high property taxes per capita, populations exceeding one million. Cornell used the standard error of estimate expressed in relative terms and the coefficient of dispersion as tests of goodness of fit.

Recognizing that the logarithmic formula might not satisfy his conditions of simplicity and common sense appeal, Cornell developed an alternative method in linear form. A straight averaging with equal weight of the six factors and the criterion was tried and found unsatisfactory. Separate regression formulas were then determined for each of the six economic measures. The distribution was broken to allow for counties with special characteristics. The resulting six predictions were then averaged to obtain the index of taxpaying ability. Largely because the counties were grouped into four categories, this method produced a smaller relative error than the logarithmic formula. An application of this technique was made by McLure[19] to the state of Mississippi in a survey in 1948.

Cornell sought to overcome the obstacles inherent in the least squares method by expressing all variables as logarithmic functions, by averaging the predictions from six separate regression lines, and

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[19] William P. McLure, Let Us Pay for the Kind of Education We Need.

by empirically breaking the distribution. The logarithmic formula was admittedly lacking in administrative feasibility because of its complexity and the abundance of empirical adjustments. This formula may have been useful as a basis for developing conclusions as to the reliability of refined predictive formulas. The necessity of breaking the distribution of the counties of New York sharply reduces the usefulness and objectivity of both the logarithmic formula and the linear regression average formula, because independent workers would not obtain the same results. Further, the effect of intercorrelations was ignored in the latter technique. If the index of taxpaying ability is expected to have continued practical use, it must be administratively feasible and potentially palatable to the lawmakers. No legislature has yet accepted an index developed from a broken distribution of local administrative units.

Johns

The first economic index of taxpaying ability written into a state law was developed by Johns[20] in 1939 for the state of Alabama. He made considerable use of Cornell's studies but rejected the techniques recommended by Cornell for determining the weights of factors included in the index. Johns recognized at that time that the multiple regression technique by least squares was not the proper

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[20]R. L. Johns, "An Index of the Financial Ability of Local School Systems to Support Public Education." (Mimeographed Report)

mathematical instrument for developing weights, and felt that an index could be developed by empirical methods that would be sufficiently reliable for practical use. The formula for Alabama was developed with a minimum use of involved statistical methods and an emphasis on simplicity.

His method was essentially a technique of weighted averages. The criterion and all the independent variables were expressed as percentages of the state total, in the same manner as Mort and Cornell. While Johns did not define his work in mathematical terms, the following formula essentially describes the method he employed. Let X_p equal the predicted taxpaying ability of a county; $X_1, X_2, \ldots X_n$ equal the independent variables used in the formula; and $A_1, A_2, \ldots A_n$ equal the coefficients or weights assigned to the variables. The general formula would then be

$$X_p = A_1X_1 + A_2X_2 + \ldots A_nX_n.$$

He had no mathematical method for determining the respective values of $A_1 \ldots A_n$, and therefore he simply estimated these weights by trial and error process. The following two tests were employed in determining the best weights of the factors: (1) correlation of the index with the criterion, and (2) average relative deviation of the index from the criterion. The statistical worker will at once recognize that this method has some serious limitations because no two independent workers would arrive at the same weights except by accident, and that probability would be very small. Nevertheless, this

technique did overcome the limitations of the least squares technique because, as will be demonstrated later, this technique produced formulas that fit the criterion better than the formula resulting from use of the least squares method. Further, it is just as sensitive to the small counties as it is the large counties, and it will give a prediction within the range of evidence given by the economic factors.

In recent years this technique has been applied to other southern states. Johns developed the indices of taxpaying ability adopted by the legislature of Florida in 1947, and recommended in the South Carolina Survey[21] of 1948. Johns and Cornell assisted in calculating an index of taxpaying ability for Georgia using the empirical weighting methods developed by Johns.[22] The index of ability adopted by Texas[23] in 1949 appears to be a result of this same technique.

The ultimate selection of economic factors must of course be governed by the characteristics of the state involved. It is seen in Table 1 that many of the economic factors used in these southern states are similar in nature to those employed by Mort and Cornell. The weights of the factors are expressed as per cents in order that

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[21] Public Schools of South Carolina, pp. 321-24.

[22] A Survey of Public Education of Less than College Grade in Georgia. A Report to the General Assembly of Georgia by its special committee on Education. Atlanta, 1947.

[23] Texas School Law, 1949.

Table 1

Indices of Taxpaying Ability for Five Southern States

Factors	Ala.[24]	Fla.[25]	Ga.[26]	S.C.[27]	Texas[28]
Assessed Valuation	.500	.250	.316		.200
Retail Sales	.176	.200	.105	.361	
Auto Registrations	.147	.150	.105	.164	
Farm Income	.029	.050		.056	
Public Utilities	.088		.105	.168	
State Income Tax	.029		.053		
Effective Buying Power		.300	.316		
Employed Workers				.251	
Value Added by Manuf.	.029				
Railroad & Tel. Prop.		.050			
Population					.080
Income[29]					.720
Totals	1.000	1.000	1.000	1.000	1.000

all the formulas for the various states may be contrasted.

Because no other acceptable measure was available, assessed valuation of property served as the statistical criterion for developing formulas for determining taxpaying ability in Alabama, Florida,

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[24] Alabama School Law. Issued by Board of Education, 1941, p. 95.

[25] Florida Statutes, 1947. Committee Substitute for house Bill no. 146, "An Act Relating to Education." p. 79.

[26] A Survey of Public Education....In Georgia, p. 294.

[27] Public Schools of South Carolina, p. 323.

[28] Texas School Law, pp. 9-10.

[29] Ibid. This measure of income was obtained by summing value added by manufacture, value of mineral products, farm products, retail establishment payrolls, wholesale establishment payrolls, and service establishment payrolls.

Georgia, and South Carolina. It should be noted that assessed valuation was given a weight of .500 in the Alabama index, of .316 in the Georgia index, of .250 in the Florida index, and of .200 in the Texas index. Inclusion of assessed valuation as an economic factor may be justified as a temporizing expedient that was necessary before a full departure from assessed valuation could be established. This practice should be discontinued with the growth of confidence in the index of financial ability.

The question has not been answered as to how much more accuracy can be obtained by employing more refined methods of determining the weights of the factors involved in the index. However, the primary objection to this empirical technique is its lack of objectivity. As Johns well recognized, independent workers could not obtain the same results by following his technique because it is neither standardized nor mathematical. The practical value of an instrument such as the ability index is measured by its ultimate use. The development of administratively feasible indices that have been accepted makes the contribution of Johns particularly significant. The fact that these indices are being used, and that the demand for this type of measure is growing, emphasizes the need for a more standardized technique. Johns has long realized the desirability of using a mathematical technique, but no formula has yet been developed which meets the standards set up by Johns and the present writer. When techniques more standardized and objective are developed, their

potential use will be materially increased as a result of his and Cornell's investigations.

CHAPTER III

SELECTION OF A FORMULA WHICH MEETS THE STANDARDS

The nature and characteristics of the desired formula can be derived from the standards governing the development of the index. These standards, set forth in Chapter II, are as follows:

1. The index and all economic factors should be objective; therefore all data pertaining to these factors should be obtainable from reliable published sources.

2. All economic factors and the index should be independent from the influence of local assessing bodies.

3. Each economic factor should measure some different aspect of the wealth of the state, and a sufficient number should be included in order to represent all the principle elements of the wealth of the state.

4. The index should be based on some validating measure which directly corresponds to the actual value of property.

5. The mathematical formula employed for the development of the index of taxpaying ability should be as sensitive to the small local units as it is to the large local units in predicting relative ability.

6. The index of taxpaying ability should be as equitable as possible without undue complexity in order that the formula be administratively feasible.

Type of Function Desired

The type of formula that will meet the standards given above is a function that will combine the selected economic factors with weights that assess their relative influence on the criterion. This function can be expressed as

(1) $X_p = A_1X_1 + A_2X_2 + A_3X_3 + \ldots A_nX_n$,

and might properly be called a weighted average. This is the same type of function employed in the formulas described in Table 1.

It was demonstrated in Chapter II that a formula which permitted a prediction outside the range of evidence given by the economic factors was insensitive to the local units of small wealth and therefore inequitable. Hence, one of the characteristics of the desired formula is that all predictions must be within the range of the evidence. This requirement is satisfied in the general form (1) as long as the weights $A_1, \ldots A_n$ are positive. It should be noted that the equation described excludes the constant term so often found in regression equations. This exclusion is a necessary condition to the fulfillment of this requirement.

If the desired formula is to measure the ability of small local units as accurately as large local units, then the test of goodness of fit with the criterion must be expressed in terms of relative deviations (residuals) rather than actual deviations. In other words, the percentage of error in the predicted value from the best observed estimate of actual value would be a minimum. It does

not matter whether the percentage of error in each case is positive or negative. The mathematical expression of this test is given by

$$(2) \quad \sum \frac{\left| X_p - X_o \right|}{X_o},$$

meaning the absolute sum of the relative residuals. Several attempts have been made to develop mathematically a regression equation based on this property. In discussing various types of residuals, Wolfenden[1] mentions the work of Edgeworth:

> Edgeworth's method, which was suggested in 1887, would minimize the sum of the absolute values of the residuals, and thus is based on 'LaPlace's First Law of Error,' i.e., on the hypothesis that the error function is of the form $y = \frac{k}{2} e^{-k|x|}$ where $|x|$ is taken positively in both directions, so that the median is the most probable value. The solution for the case of two unknowns is explained and illustrated by Bowley;[2] when there are more than two unknowns, however, it is difficult to minimize the sum of the absolute values of the residuals, because, when all the terms are to be taken as positive, it is not known until the work is completed which are naturally positive or negative.

Pearl[3] suggested a way of modifying the application of the method of least squares in order to compensate for changes in the probability of the occurrence of an error of any given magnitude. Instead of using the sum of the squares of the residuals, he employed the sum of the squares of the relative residuals in developing the regression equation. This residual was the difference between the

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[1]Hugh Herbert Wolfenden, The Fundamental Principles of Mathematical Statistics, pp. 171-72.

[2]A. L. Bowley, F. Y. Edgeworth's Contributions to Mathematical Statistics, pp. 103-109.

[3]Raymond Pearl, Studies in Human Biology.

predicted value and the observed value divided by the observed value. If the desired measure expressed by (2) were squared, it would be identical to Pearl's test.

Hendricks[4] suggested a similar modification in order to incorporate the use of the relative residual. He used the difference between the predicted and observed values divided by the predicted value. Using one independent variable, Hendricks developed regression equations by the method of least squares, by Pearl's modification, and by his own modification. The results of these experiments indicated that his procedure was slightly superior to Pearl's, and in turn both procedures were superior to the prediction from a regression equation by the method of least squares. The data of the experiment were such that the magnitude of the error increased with the magnitude of the variables. Hendricks did not attempt to apply this procedure to a predictive equation involving multiple independent variables. It should be made clear that both these modifications yielded equations where the sum of the squares of the relative residuals was a minimum. Results of several investigations on the part of this writer indicate that these modifications advanced by Pearl and Hendricks are not applicable in a practical way to multiple regression equations. Even if they were, such a technique still would not meet precisely the test expressed in (2).

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[4] W. A. Hendricks, "The Use of the Relative Residual in the Application of the Method of Least Squares," pp. 458-68. Annals of Mathematical Statistics. 1931, Vol. II.

Correlations and Intercorrelations

It has been established that the coefficients of the weighted average in (1) must always be positive. The factors selected should also show a positive simple correlation with the criterion. The more closely a particular economic factor is related to the criterion the greater should be its influence in determining the prediction, provided due consideration is given to its interrelationship with other economic factors included in the formula. The concern of this study is with a technique that will determine the per cent of influence each factor exerts on the criterion. When the relative influence of each factor is determined satisfactorily the desired weights for the formula are obtained.

Although it is desirable for each factor to have a positive simple correlation with the criterion, it does not follow that the factor with the greatest coefficient of correlation should have the greatest weight. The effect of intercorrelations must also be considered. A factor may show a very high simple correlation only because of its closer relationship to another highly correlated variable. For example, consider the two factors, population and gainfully employed workers. Both measures might be expected to correlate highly with property valuation. Both factors measure a similar aspect of wealth, and therefore would be expected to correlate highly with each other. The question is which measure has more influence on the value of property. That could be better determined

by the correlation of each measure with the criterion when the net effects of other considered factors have been removed from the criterion. If the technique for developing the weights does not take into consideration the effect of intercorrelations, one could load up an equation with highly correlated similar aspects of wealth, such as number of children, number of men, total population, gainfully employed workers and other population measures. Most of the various economic factors are positively related to each other, and our problem is to ascertain what portion each factor's influence is due to itself and what portion is explained by the relationship with other factors.

Summary

A weighted average formula with the characteristics described in this chapter will satisfy the given standards. An objective technique for calculating such a formula has not been adequately developed in the literature. The weighted average function herein described is essentially the same function that has been sought and tried in some of the southern states mentioned in Table 1. To summarize, the formula that meets the standards set forth in this study should have the following characteristics:

1. The function is a weighted average, and hence will contain no constant term.

2. The respective weights are positive.

3. All predictions are within the range of evidence given by the selected factors.

4. The sensitivity to the local units of small ability is reflected in the test of goodness of fit, viz., the absolute sum of the relative residuals is a minimum.

5. The effect of intercorrelations is considered in arriving at the proper weights.

6. The respective weights will represent the per cent of influence each factor exerts on the criterion.

7. The technique should be objective enough that it can be applied without the necessity of making empirical adjustments in order that independent workers may be able to obtain identical results from the same data.

CHAPTER IV

SELECTION OF A STATISTICAL CRITERION FOR THE DEVELOPMENT OF THE FORMULA

If property were assessed throughout a state at its full value or at a uniform per cent of its full value, assessed valuation would be a satisfactory measure of ability of the local administrative units to support education. It is well known that this condition does not exist in any state where assessments are made locally. Since few states have set up central tax bodies to determine full value of property, the use of an economic index to estimate true taxpaying ability has gained a wider acceptance, as cited in Chapter II. The financial ability of a state to support education is different from the ability of the local administrative units. The state has the privilege of taxing all types of wealth, whereas local school units with few exceptions are limited to the property tax. In 1926 Mort[1] stated:

> Happily, the problem may be made a simple one. It may be simplified first by eliminating theoretical ability to pay from consideration. We need only determine ability to pay under the tax system actually to be used. Since we must deal with communities which have no power over their ability as it would be under an ideal tax system. To build our system of state aid on such a foundation would throw excessive burdens upon the actual taxpayers in some

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[1] Mort, State Support for Public Schools, pp. 16-17.

communities, simply because there happened to be wealth in those communities that was not taxable under the existing system of taxation.

Another step in simplification is introduced by the necessity of considering the ability to pay local taxes apart from the ability to pay state taxes. We are interested first in the ability of each community to support its education program through local taxes. For this purpose, we need a measure of the sources which the tax system permits the localities to tap.

If it were necessary to obtain a single measure of ability to pay taxes, state, local, and federal, we should be faced with the difficult task of weighting the tax sources according to the fair rates of returns. We should have to meet a similar situation in weighting different sources for local revenues if it were not for the fact that local school revenues are derived almost entirely from property taxes. As a matter of fact, we need only to obtain the value of taxable property to have a measure of the relative ability of a locality to pay local taxes for supporting schools. This cannot be done except where assessment of property values is made by a centralized system of assessment under state control, or where an adequate state agency is established for determining the rates at which local officers assess property. In the former case, the assessed value of property is the measure of ability to pay local taxes for school purposes. In the latter, either the equalized or the full value obtained by applying the rates of assessment to assessed values is the desired measure.

Need for Criterion

In the development of any predictive formula, it is necessary to select one observed measure of what is to be predicted as the validating measure. In this study, the validating measure is referred to as the criterion. The weights of the other measures used in a composite index are determined by their influence and relationship with this validating measure. According to Mort's statement just quoted, the best single measure of local ability is the true value

of taxable property, generally determined by a centralized tax body. It should be realized that even this measure is imperfect, because of the fallible procedures used to determine true value of property.

Proposed Measures

Cornell[2] used the true value of real property as determined by the State Tax Commission as the criterion for his index of the tax-paying ability of the counties of New York State. The use of aggregate taxes on real property was suggested by the same author as an alternative criterion, because of its very close relationship with full valuation of property. Assessed valuation was not recommended by Cornell[3] as a criterion for two reasons: (1) the average error of a formula based on assessed valuation as the criterion was over 21 per cent greater than that obtained from an "acceptable" criterion; (2) no statistical improvement in New York resulted from a formula based on assessed valuation over assessed valuation itself. Cornell's rejection of assessed valuation as a criterion is by no means conclusive, but merely demonstrated the superiority of true valuation. As stated in Chapter II, assessed valuation as a criterion has been judged fairly satisfactory in practice where nothing better was available.

The technique of developing an index advanced by the writer

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[2]Cornell, op. cit., pp. 15, 64-65.

[3]Ibid.

will be applied later in this study to the states of South Carolina and Tennessee. For South Carolina, Alabama, and Florida assessed valuation was taken as the best available criterion; for Tennessee the best available criterion was an estimate of true value of real property based on the ratio of assessed value to selected actual sales.

Summary

In any state, the ultimate selection of a criterion on which to develop an economic index depends on the available data. It would seem that some type of estimate of the real value of property, even though imperfect, represents the best criterion of taxpaying ability. The use of aggregate taxes on property as a criterion is open to several objections. Its validity is questionable, it is subject to local manipulation, it is dependent upon state practice with respect to taxing property, and it rewards a local unit for inefficient collection of taxes. Although assessed valuation by definition corresponds to actual property values, the immediate absence of a better estimate does not automatically guarantee its satisfactory use as a criterion. In Chapter VII further suggestions are given for the selection and development of a suitable criterion.

CHAPTER V

ANALYSIS OF ECONOMIC FACTORS TO INCLUDE IN THE FORMULA FOR THE INDEX

The purpose of this study is not to propose new techniques for the analysis of economic factors to include in the predictive formula, but to propose a technique for weighting a given set of factors. For illustrative purposes, the factors used in developing a weighting technique should be relatively sound. Therefore, a brief review of what has been done in factor analysis serves as a guide in the selection of economic factors to be employed in this study.

Types of Factors

Cornell[1] divided into three categories the possible measures that might be used in a formula to predict taxpaying ability in New York:

A. Direct Measures of the Value of Real Property
   1. Value of farm real estate
   2. Rental of homes
   3. Value of owned homes
   4. Value of construction business

B. Measures of Population
   1. Total population
   2. Population density
   3. Urbanization
   4. Proportion of agricultural population

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[1]Cornell, op. cit., pp. 19-32.

5. Rate of population increase
6. Birth rate
7. Proportion of population of various ages

C. Measures of Income and Purchasing Power, Resultants and Concomitants
1. Net taxable income
2. Value of manufacturing, mining and farm products
3. Retail sales
4. Number of gainfully employed workers
5. Residence telephones
6. Savings deposits
7. Postal receipts
8. Motor vehicle registrations
9. Aggregate taxes on property

Cornell used the following six of these measures in developing indices for the counties of New York: total population, retail sales, motor vehicle registrations, production, income tax returns, and postal receipts.[2]

Mort[3] classified possible measures of the taxpaying ability of the states under the following aspects of wealth:

A. General
1. Income tax returns
2. Population
3. Bank resources
4. Time deposits
5. Postal receipts
6. Motor vehicle registrations

B. Manufacturing and Industry
1. Factory wages
2. Value of manufactured products
3. Value added by manufacture
4. Factory wage earners

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[2] Ibid., p. 48.

[3] Mort, Federal Support for Public Education, pp. 180-84.

5. Horsepower in manufacture
6. Production of electric power
7. Employment: all industries

C. Business Activity
1. Commercial failures: total liabilities
2. Stock transfers
3. Bank suspension deposits
4. Gasoline consumption
5. Retail trade--net sales
6. Wholesale trade--net sales
7. Value of construction business

D. Agriculture
1. Farm cash income
2. Farm gross income
3. Value of farm real estate

E. Natural Resources
1. Petroleum production
2. Value of mineral products

The factors selected by Mort in developing his index of the relative ability of the states have already been enumerated.

The factors that were employed in the indices developed by Johns appeared in Table 1. As can be seen in those states observed, it is not expected that many states should use exactly the same combination of economic factors. Selection of factors of necessity must be geared to the nature of a state's economy, and must be governed by the availability of data. It is apparent that the measures of natural resources in category E would not apply to those states where such resources are insignificant.

The problem may arise as to how many factors should be included in the formula. There are several statistical methods that can be used in factor analysis to determine whether additional factors

should be used. On the basis of beta coefficients, Cornell eliminated two factors from his logarithmic formula because of an insignificant contribution.[4] This is one of the most common measures of the relative influence of independent variables.

Factors Employed in This Study

The measures of ability adopted in this study were selected from those employed by Johns[5] in his proposed index for the counties of South Carolina. These measures (retail sales, auto registrations, farm income, public utilities, gainfully employed workers) satisfy fairly well the standards governing factor selection. In order to prevent overlapping of factors and to make more sensitive the selected factors, agricultural workers and government workers have been subtracted from the factor gainfully employed workers. Farm workers are considered in farm income, and since governmental enterprises are non-taxable, government workers should not be included in this measure of ability. The sources of data for the five above-mentioned factors were respectively Sales Management Magazine, State Highway Department, United States Census, State Tax Commission, and United States Census.[6] These sources are accepted as reliable, and establish the respective factors as both objective and free from local influence. Accepting Mort's classification of measures, at

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[4] Cornell, loc. cit.

[5] Public Schools of South Carolina, p. 323.

[6] Ibid.

least one of the above factors is represented in each of the first four categories outlined by Mort. Therefore the standard is satisfied that requires the selected economic factors to measure different aspects of the state's wealth.

CHAPTER VI

TECHNIQUES FOR DEVELOPING THE WEIGHTS OF THE ECONOMIC FACTORS

In Chapter II standards for the development of an index of tax-paying ability were set forth, the least squares method was examined, and evaluation was made of some previous studies. The desired type of formula and its characteristics were described in Chapter III, criteria were discussed in Chapter IV, and a brief review of the selection of economic factors was made in Chapter V. Several techniques of combining and weighting a given set of factors will be examined in this chapter and evaluated in the light of the given standards and characteristics.

In selecting the proper combination of economic factors, it is well to observe the nature of their relationships with the criterion. It is recognized that the relationships of each factor with the criterion may not be strictly linear. Actually, the measures of farm income and automobile registrations generally have a greater relative value in districts of low ability than in wealthy districts, and therefore increase at a slower rate than property values. On the other hand, the measures of retail sales and gainfully employed workers generally increase at a greater rate than property values. Hence a desirable element of balance is attained when at the extremities of the distribution some factors tend to overpredict while others tend to underpredict. These four factors have been selected as the measures on which various formulas for the counties

of Tennessee are developed and examined. This state has been chosen for illustrative purposes because a usable criterion is available in the form of an analysis of actual sales. Since we are concerned only with relative taxpaying ability, all data are expressed as per cent, unless otherwise specified.

Broken Distribution

The disparity between the concentration of wealth in the more populous areas and the relative poverty in the sparsely settled rural areas of a state is cause for concern in dealing statistically with the distribution of all the counties. It is evident that a few metropolitan areas can exert undue influence in the calculations of weights for all the localities, unless corrective measures are taken. It is also apparent that a better fit could be obtained if the distribution were broken one or more times and separate formulas were developed for each group. Cornell was able to obtain an excellent fit with the criterion by employing a broken distribution.[1] Desirable as this result may be, any technique involving a broken distribution violates the standards of objectivity and administrative feasibility.

A Method Involving Relative Residuals

In an effort to incorporate relative residuals in approximating the relative influence of the factors on the criterion, the following

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[1] Chapter II, p. 27.

formula was developed to calculate the respective weights:

$$A_1 = \frac{\dfrac{1}{\sum \dfrac{|X_1 - X_0|}{X_0}}}{\dfrac{1}{\sum \dfrac{|X_1 - X_0|}{X_0}} + \cdots\cdots\cdots + \dfrac{1}{\sum \dfrac{|X_4 - X_0|}{X_0}}}$$

In like manner A_2, A_3 and A_4 are determined, where

X_0 = the criterion--the best estimate of true property value

X_1 = auto registrations

X_2 = sales tax collections

X_3 = gainfully employed, non-agricultural, non-government workers

X_4 = farm income

Assuming that the more closely a factor X_n is related to the criterion X_0 the greater its relative weight should be, the absolute sum of the relative deviations of the most important factor will be least, and vice versa. Therefore, reciprocals were used in order to arrange the values in the order of their importance. The division of each reciprocal by the total of the reciprocals served the necessary function of adjusting the weights so as to total unity.

When data for the counties of Tennessee were used to calculate the weights A_1, A_2, A_3, and A_4 the formula predicting relative ability was found to be

$$X_p = .333X_1 + .278X_2 + .296X_3 + .093X_4.$$

The average per cent deviation of the prediction from the criterion was 22.0, which compared favorably with the results of many other

techniques.[2] Since the influence of intercorrelation was not reckoned with this technique is not satisfactory.

Grouped Homogeneous Equations

A rather simple technique was examined whereby the data of the local administrative units were grouped homogeneously and combined in such a way as to yield the same number of equations as independent variables. Then the unique solution of the simultaneous linear equations would give the desired weights. This plausible method was applied to the 46 counties of South Carolina which were divided into five groups arranged in the order of their assessed valuation. The five factors used in South Carolina were public utilities, farm income, auto registrations, retail sales, and gainfully employed workers. The solution of the five equations gave the following weights:

$$X_p = .047X_1 + .266X_2 - .399X_3 + .237X_4 + .849X_5.$$

The fit of this formula with the criterion was poor, as would be expected. An examination of the five equations developed revealed that both the coefficients and the constants in each of the equations were almost proportional. This meant that the five hyper-planes represented by the five equations approached coincidence. Little faith is placed in the solution of such equations when the data represent measurements, since a small error would result in a

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[2] Chapter VII, Table 3, formula II.

significant change in the solution. This formula would be further disqualified on the basis of the negative coefficient.

Beta Coefficients

The use of Beta coefficients[3] has been offered as a means of measuring the separate effect of independent variables on the dependent variable. The regression coefficients can be made more comparable if each variable is expressed in terms of its own standard deviation. Therefore, Beta coefficients are nothing more than simple adjustments made to the coefficients of the multiple regression equation. If $b_1, b_2, \ldots b_n$ represent the regression coefficients of factors $X_1, X_2, \ldots X_n$, then the resulting Beta coefficients $A_1, A_2, \ldots A_n$ can be expressed in this manner:

$$A_1 = \frac{\sigma_1 b_1}{\sigma_0}, \qquad A_2 = \frac{\sigma_2 b_2}{\sigma_0} \qquad A_n = \frac{\sigma_n b_n}{\sigma_0}$$

When weights were calculated by this procedure for the counties of Tennessee and adjusted to total unity, this formula resulted:

$$X_p = .080X_1 + .318X_2 + .539X_3 + .063X_4.$$

The test of goodness of fit, however, was not impressive, since it showed that the average of the relative residuals was 25.5 per cent.[4] It can be seen from the above definition that the Beta coefficients would be negative whenever the regression coefficients are negative.

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[3] Mordecai Ezekiel, Methods of Correlation Analysis, p. 183.

[4] See Chapter VII, Table 3, formula IV.

Hence this instrument does not meet the desired characteristics.

Coefficient of Part Determination

The coefficient of part correlation was developed in 1926 by Ezekiel and Smith[5] as a device for measuring the separate effect of individual variables on the dependent variable. Mort[6] investigated the coefficient of part correlation in 1936 as a possibility in determining the relative ability of the states. The measure applies to the variables in a linear multiple regression equation determined by least square methods. This coefficient measures the amount of correlation between each independent variable and the variation remaining in the dependent variable after the estimated effects of the other independent variables have been eliminated.[7] The symbol expressing this correlation is 01^r234. If the multiple regression equation with four independent variables is expressed by

(1) $$X_p = b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + k.$$

then

01^r234 = correlation between X_1 and $(X_p - b_2X_2 - b_3X_3 - b_4X_4)$.

The coefficient of part determination is defined simply as the

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[5] B. B. Smith, "Correlation Theory and Method Applied to Agricultural Research." U. S. Department of Agriculture. pp. 57-60.

[6] Mort, Federal Support for Public Education, p. 314.

[7] Ezekiel, op. cit., pp. 181-83.

square of the coefficient of part correlation. The computation of such coefficients is facilitated by the formula[8]

$$(2) \qquad 01R^2 234 = \frac{b_1^2\sigma_1^2}{b_1^2\sigma_1^2 + \sigma_0^2(1 - R_0^2.1234)} ,$$

where R represents the multiple correlation coefficient and σ_1 represents the standard deviation of the X_1 distribution.

Ezekiel gives the following explanation as to exactly what the coefficients of part determination measure: "....they may be explained as measuring the proportion of variance remaining in the dependent variable after the net effects of the other variables are taken account of, which can be explained by adding the additional factor."[9] As an illustration, if $03r^2 124$ were .368, it would mean that X_3 would account for 36.8 per cent of the variance in X_p when the other variables are taken account of. The fact that the relative influence of the factors can be expressed as per cents suggests that they are comparable to each other. As to how useful coefficients of part determination are in practice can be shown when this measure is applied to the 95 counties of Tennessee.

In the basic formula for predicting relative ability,

$$(3) \qquad X_p = A_1X_1 + A_2X_2 + A_3X_3 + A_4X_4,$$

if both sides of the equation are summed, then

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[8] Ibid., This formula is derived on pp. 379-80.

[9] Ibid., p. 183. Proof of this relationship is given on pp. 375-76.

$$(4) \qquad \Sigma X_p = A_1 \Sigma X_1 + A_2 \Sigma X_2 + A_3 \Sigma X_3 + A_4 \Sigma X_4.$$

Since ΣX_p means the sum of the percentages of the state's property values for all counties, its value must be 100 per cent or 1. In like manner, the values ΣX_1, ΣX_2, ΣX_3, and ΣX_4 are also unity. Therefore, equation (4) becomes

$$A_1 + A_2 + A_3 + A_4 = 1.$$

Since it is apparent that the weights should sum to unity, every combination of weights by any technique should be adjusted in order to meet this requirement. If this adjustment is not made, it is necessary to divide the index of each county by the total in order to determine its relative ability in per cents. The former procedure is decidedly preferred. Then the respective weights, A_1 can be defined in terms of coefficients of part determination. For example,

$$A_4 = \frac{04r^2 123}{01r^2 234 + 02r^2 134 + 03r^2 124 + 04r^2 123}$$

Using the four selected factors for the state of Tennessee, the index of relative taxpaying ability obtained by use of coefficients of part determination gives

$$(5) \qquad X_p = .163X_1 + .350X_2 + .369X_3 + .118X_4.$$

Table 2 shows the criterion, the value of each factor, and the computed index for all of the 95 counties of Tennessee. Experiments by the writer with numerous formulas and techniques led to the actual testing for goodness of fit of more than a score of formulas. The results showed the average per cent deviations ranged from 22.0 per

TABLE 2

AN INDEX OF THE TAXPAYING ABILITY OF TENNESSEE COUNTIES

BASED ON COEFFICIENTS OF PART DETERMINATION

County	Per Cent of State Total					
	Full[10] Value of Property	Auto[11] Registrations	Retail[12] Sales	Gainf'y[13] Employed Workers	Farm[14] Income	Index of Relative Ability
Anderson	.564	2.044	1.326	.840	.189	1.130
Bedford	.934	.984	.609	.801	1.546	.852
Benton	.187	.294	.162	.179	.363	.214
Bledsoe	.189	.164	.103	.126	.414	.158
Blount	2.045	1.873	1.530	1.532	.845	1.506
Bradley	.789	1.313	.881	1.060	.638	.989
Campbell	.335	.693	.681	.985	.277	.747
Cannon	.220	.234	.109	.102	.479	.170
Carroll	.508	.821	.558	.477	1.238	.651
Carter	1.374	1.002	1.041	1.192	.383	1.013
Cheatham	.266	.251	.111	.155	.596	.207
Chester	.208	.369	.199	.161	.597	.260
Claiborne	.526	.438	.278	.374	1.266	.456
Clay	.122	.150	.074	.096	.391	.132
Cocke	.400	.616	.382	.418	1.013	.508
Coffee	.441	.664	.514	.532	.659	.562
Crockett	.546	.427	.280	.187	1.821	.451
Cumberland	.424	.391	.327	.303	.228	.317
Davidson	12.389	11.180	15.130	14.160	2.031	12.583
Decatur	.152	.244	.165	.155	.400	.202

[10] Data compiled by the railroads in each county based on actual sales of real estate for the year 1945.

[11] Department of Finance and Taxation, State of Tennessee. Motor Vehicle Division, Auto Registrations. 1947.

[12] Ibid. Sales Tax Collections. 1947.

[13] U. S. Bureau of the Census, Sixteenth Census of the United States: 1940. Population.

[14] Census of Agriculture, 1945. Bureau of Agricultural Economics, U. S. Department of Agriculture.

Table 2 (Continued)

County	Per Cent of State Total					
	Full Value of Property	Auto Regist-rations	Retail Sales	Gainf'y Employed Workers	Farm Income	Index of Relative Ability
DeKalb	.327	.263	.144	.149	.758	.238
Dickson	.353	.552	.351	.526	.723	.492
Dyer	1.097	1.051	1.062	.883	2.267	1.136
Fayette	.526	.594	.292	.229	2.066	.527
Fentress	.230	.191	.175	.265	.157	.209
Franklin	.621	.717	.521	.623	1.063	.655
Gibson	1.227	1.611	1.164	.990	3.192	1.412
Giles	.853	.836	.565	.587	2.256	.817
Grainger	.263	.296	.099	.141	.883	.239
Greene	1.678	1.359	.817	.719	3.154	1.145
Grundy	.228	.201	.184	.254	.168	.211
Hamblen	.641	.935	1.064	.589	.934	.852
Hamilton	7.123	6.388	9.311	9.924	.389	8.008
Hancock	.219	.138	.058	.063	.695	.148
Hardeman	.368	.449	.326	.373	1.235	.471
Hardin	.382	.339	.247	.334	.649	.342
Hawkins	.727	.619	.388	.510	1.474	.599
Haywood	.540	.551	.399	.356	2.317	.634
Henderson	.230	.471	.302	.248	1.097	.403
Henry	.673	1.000	.581	.683	1.085	.746
Hickman	.406	.314	.158	.263	.521	.265
Houston	.110	.125	.063	.122	.152	.105
Humphreys	.295	.281	.194	.227	.492	.256
Jackson	.253	.212	.105	.126	.728	.204
Jefferson	.519	.617	.287	.378	.955	.453
Johnson	.311	.244	.139	.163	1.120	.281
Knox	8.386	7.551	10.314	9.060	1.332	8.341
Lake	.427	.314	.254	.198	1.643	.407
Lauderdale	.686	.562	.549	.356	2.115	.665
Lawrence	.617	.804	.555	.553	1.276	.680

Table 2 (Continued)

County	Per Cent of State Total					
	Full Value of Property	Auto Regist-rations	Retail Sales	Gainf'y Employed Workers	Farm Income	Index of Relative Ability
Lewis	.135	.196	.120	.119	.090	.128
Lincoln	.920	.811	.528	.567	2.261	.793
Loudon	.618	.741	.477	.726	.538	.619
McMinn	.544	.947	.783	.970	.950	.898
McNairy	.352	.478	.309	.261	.878	.386
Macon	.312	.365	.149	.151	.943	.279
Madison	1.496	1.953	2.009	2.001	1.992	1.995
Marion	.398	.470	.344	.562	.289	.438
Marshall	.718	.756	.507	.448	1.481	.641
Maury	1.569	1.472	1.127	1.521	2.559	1.498
Meigs	.089	.134	.035	.097	.254	.100
Monroe	.539	.576	.390	.460	.852	.501
Montgomery	.961	1.179	.957	1.198	1.554	1.153
Moore	.111	.138	.035	.058	.420	.106
Morgan	.136	.305	.185	.342	.142	.257
Obion	1.132	1.109	.805	.955	2.529	1.113
Overton	.270	.274	.198	.193	.316	.222
Perry	.166	.124	.063	.086	.323	.112
Pickett	.083	.075	.036	.062	.194	.071
Polk	.512	.297	.317	.385	.172	.322
Putnam	.663	.803	.621	.575	.568	.627
Rhea	.258	.522	.312	.492	.267	.407
Roane	.502	.780	.969	.923	.318	.844
Robertson	.869	.887	.523	.679	2.413	.863
Rutherford	1.191	1.407	.892	.941	2.253	1.155
Scott	.139	.315	.268	.409	.096	.307
Sequatchie	.108	.162	.077	.100	.105	.103
Sevier	.664	.681	.441	.391	1.005	.528
Shelby	19.565	15.346	20.941	20.587	3.202	17.805
Smith	.506	.440	.208	.228	1.423	.397

Table 2 (Continued)

County	Per Cent of State Total					
	Full Value of Property	Auto Regist-rations	Retail Sales	Gainf'y Employed Workers	Farm Income	Index of Relative Ability
Stewart	.169	.232	.085	.135	.405	.165
Sullivan	3.086	3.236	3.883	3.105	1.286	3.184
Sumner	1.082	1.136	.561	.779	2.331	.944
Tipton	.785	.974	.573	.408	2.510	.806
Trousdale	.224	.230	.085	.097	.642	.179
Unicoi	.268	.360	.283	.407	.207	.332
Union	.143	.250	.059	.089	.388	.140
Van Buren	.074	.066	.028	.048	.064	.046
Warren	.566	.613	.488	.479	.688	.529
Washington	1.769	1.947	2.298	1.834	1.720	2.001
Wayne	.273	.225	.163	.223	.355	.218
Weakley	.798	1.095	.499	.578	2.026	.805
White	.376	.413	.263	.338	.613	.356
Williamson	1.128	.739	.397	.575	2.213	.733
Wilson	.798	1.004	.589	.683	1.895	.845
Total	100.000	100.000	100.000	100.000	100.000	100.000

cent to 39.0 per cent. The test on formula (5), however, showed an average per cent deviation of 21.0 per cent from the criterion. The average deviation of assessed valuation from the criterion was found to be 22.8 per cent.

Evaluation of the Part Determination Technique

According to Ezekiel[15] this is the first practical application of coefficients of part determination. How well does the formula

[15] See Appendix A.

developed by this measure meet the characteristics described in Chapter III? The type of function exhibited in formula (5) is a weighted average with no provision for the occurrence of a constant. An examination of formula (2), whereby the coefficients of part determination are calculated, reveals that all elements of the numerator and denominator are positive under all circumstances. Therefore positive weights are guaranteed by this procedure. An average of positively weighted factors always gives results within the range of the factors.

The equitableness and sensitivity of formula (5) to the smaller counties as well as the larger counties is reflected statistically in the figure 21.0 per cent, lowest average relative residual of any formula tested on this data. The smallest 25 per cent of the counties had an average relative deviation of less than 26 per cent. This figure was lower than that for any other technique. Examination of those counties having the widest relative departure from the criterion shows that the criterion is not within the range of the factors in six of the seven such counties. This observation points to an error in the criterion rather than an error in the technique. Imperfections in the criterion in a few counties may be attributed to an unrepresentative occurrence of real estate sales. One of the distinctive features of this technique is the way in which the effect of intercorrelations is considered. It was explained earlier that the relationship of each independent variable with the dependent

variable is measured after the net regression effects of the other variables are eliminated. This procedure enables us to assay the unbiased influence of each factor on the relative value of property. It is well known to statisticians that the effect of intercorrelations is also considered in the multiple regression equation by the method of least squares; however, the regression coefficients do not accurately reflect the relative influence of each factor. The proportion of the variance in the dependent variable that can be explained by each factor is given by the coefficient of part determination. The adjustment of the weights to total unity modifies the "percent explanation" mentioned by Ezekiel;[16] however, it does not matter whether the predicted indices are adjusted or the coefficients of part determination. Therefore, this technique is especially adaptable to determining weights that will reflect the percentage of influence each factor exerts on the criterion. Finally, this technique is simple enough so that anyone who can compute regression coefficients, standard deviations, and multiple correlation coefficients can apply it to the problem of determining relative taxpaying ability.

To verify the excellent results obtained with the counties of Tennessee, this same technique was applied to the counties of South Carolina. The factors selected were the same, except for the addition of public utilities. Using assessed valuation as the best

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[16] Ibid.

available criterion, the following formula was developed:

(6) $X_p = .180X_1 + .007X_2 + .190X_3 + .333X_4 + .290X_5$.

The average relative residual of 11.4 per cent was the lowest figure obtained for any formula applied to this state. This additional evidence of the excellence of the fit with the criterion suggests that this technique may be equally successful in other states.

CHAPTER VII

SUMMARY AND RECOMMENDATIONS

Statement of the Problem

A satisfactory plan of state support for an adequate foundation program requires some measure of relative taxpaying ability of the local administrative units. Since the property tax is generally the only means available for local school funds, the measure of ability must correspond with local ability to raise property taxes. Therefore, the ideal measure would be the actual value of taxable property. True value of taxable property cannot be determined precisely, especially since it fluctuates according to the law of supply and demand. Hence the problem resolves into determining the most desirable approximation of true value of property. Where appraisal of property value is left solely to the local administrative units, such estimates of property valuation are known to be inconsistent and subject to manipulation. One solution to this problem has been the use of an economic index of relative ability. Investigations in this study have been directed toward improving the technique for developing such formulas according to certain given standards.

Findings

Previous research projects have shown that economic indices of relative ability have been developed that were superior to assessed

valuation as a measure of ability. Various techniques of developing such formulas have been examined in this study. Most of these techniques were not sufficiently objective or standardized to be applied to other states. Many techniques depended upon empirical adjustments and empirical procedures to achieve a satisfactory relationship between the prediction and the criterion. Lack of administrative feasibility describes the shortcomings of other techniques.

The use of coefficients of part determination as a technique to determine the respective weights of the factors satisfies the requirements and characteristics of a good index[1] better than any other known techniques. This technique (1) is sufficiently objective to enable independent workers to obtain the same results, (2) always predicts within the range of the economic evidence, (3) is administratively feasible, (4) is not subject to manipulation by local authorities, and (5) is more equitable and sensitive to the local administrative units than any other technique examined. A comparison of various weights and the goodness of fit of a number of formulas is shown in Table 3 and Table 4. The technique proposed in this study was applied to the data of two southern states; in one of these states assessed valuation was used as the criterion, in the other the estimated true value of property was the criterion. The fact that this recommended technique produced the closest fit in two states

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[1]These attributes were discussed in Chapter III.

Table 3

COMPARISON OF VARIOUS FORMULAS DEVELOPED TO PREDICT

RELATIVE ABILITY OF THE COUNTIES OF TENNESSEE

Factors	I*	II	III	IV	V**
Auto Registrations	.163	.333	.400	.080	.100
Sales Tax Returns	.350	.278	.250	.318	.287
Employed Workers	.369	.296	.300	.539	.504
Farm Income	.118	.093	.050	.063	.200
Av. % Deviation	21.0	22.0	23.9	25.5	30.6

*By Coefficients of part determination.

**This formula is the multiple regression equation by the method of least squares and should include the constant - .097. If this constant is multiplied by .95 and added to the other weights the sum will be unity.

Table 4

COMPARISON OF VARIOUS FORMULAS DEVELOPED TO PREDICT

RELATIVE ABILITY OF THE COUNTIES OF SOUTH CAROLINA

Factors	I*	II	III	IV	V	VI
Public Utilities	.180	.200	.168	.157	.101	.206
Farm Income	.007	.000	.056	.006	.045	.042
Auto Registrations	.190	.270	.164	.168	.293	.211
Retail Sales	.333	.230	.361	.375	.268	.280
G'fully Empl. Work's.	.290	.300	.251	.295	.293	.261
Av. % Deviation	11.4	11.8	11.8	11.6	11.9	12.2

*By Coefficients of part determination used.

differing not only in the criterion employed but in the distribution of their concentrated wealth areas is encouraging. The mathematical properties of this technique are such that good results should be obtained in any state where reliable data are available.

Limitations

It remains to be seen whether the findings of this study when applied to other states would yield a closer fit with the criterion than other objective methods. The results of observations in our two experimental states suggest the likelihood of success in other states. It would be presumptuous to deny the possibility that other techniques, as yet undiscovered, may be developed that would reveal even greater equity in predicting relative ability.

Considerable importance has been attached to the goodness of fit with the criterion. In a state such as Tennessee where the criterion was the value of property based on analysis of all sales for a year, the magnitudes of the errors of estimate of true property valuation were substantially reduced from those inherent in local assessments. Hence it could be argued that an indefinite number of successive improvements in fitting formulas to the criterion would yield more and more equitable indices of ability, provided other standards are not violated. In other words, the closer the fit, the better is the formula.

On the other hand, should the same situation prevail when

assessed valuation is taken as the criterion? A formula which by chance fits the criterion of assessed valuation perfectly would not correct for any of the variations known to be associated with this measure, and hence would be a poor formula. In developing formulas to fit this criterion closer and closer, there must be some point of diminishing deviations beyond which the formula would tend to perpetuate the inequalities inherent in assessed valuation. Calculation of this point would be a rather ambitious undertaking. Therefore, if assessed valuation happens to be used as the criterion, the degree of goodness of fit is not as important a standard as it is when a more accurate criterion of actual value is used.

Assessed Valuation as a Criterion

Some conclusions may be drawn concerning the use of assessed valuation as the criterion on which to develop the index of relative ability. The variations in assessment policy are well known; it is not an accurate measure. Because some validating measure that corresponds directly to property values is necessary to the development of an index of ability, assessed valuation has been used in several states mentioned in this study. The advantages accruing to the use of an economic index were so definite that concern over the refinements of the criterion was not paramount.

When a fairly reliable estimate of actual property values is available, as in Tennessee, a satisfactory index is expected to fit

the criterion better than assessed valuation. The proposed index for Tennessee had an average relative residual of 21.0 per cent. The availability of the estimate of actual property values in Tennessee serves as a measuring stick to examine further the suitability of assessed valuation as a criterion. Using assessed valuation as the criterion, the formula was developed by the technique of part determination, giving

(7) $\quad X_p = .280X_1 + .349X_2 + .330X_3 + .041X_4.$

Comparison of these weights with those of formula (5) in the last chapter shows considerable similarity, as one would expect. The index computed by this formula has an average relative residual of 25.1 per cent when compared to the actual value of property. When goodness of fit is measured with assessed valuation, the average relative deviation is 38.8 per cent. A comparison of assessed valuation with actual valuation for the counties of least wealth reveals a general tendency of these counties to assess at a lower ratio than the average. This tendency partially explains the average relative residual of 38.8 per cent. Since the prediction of formula (7) fits the actual value of property better than it fits assessed valuation, the conclusion might be drawn that the errors in assessed valuation are reduced when formula (7) is used to predict relative ability. On the other hand, the index obtained from formula (7) does not fit actual property values as well as assessed valuation itself fits actual property values.

When there is an appreciable explainable error in one segment of the distribution, as there seems to be in the counties of Tennessee, assessed valuation is not very suitable as a criterion. The evidence afforded by the study of the counties of Tennessee suggests that considerable improvement in the accuracy of the prediction can be obtained if some non-biased estimate of actual property value is procured. This can be achieved without transferring the power of local assessments to a central authority. It should be possible to obtain a reliable estimate of the ratio of assessments to true value from a group of dependable citizens in each county of a state. This alternative would by no means eliminate all errors, but its use would at least provide a criterion wherein the errors were distributed throughout the various localities without bias. In a state where the evidence indicated that the disparity between assessments and actual value had no relation to the size or wealth of a local unit and the error was freely distributed, assessed valuation might then be acceptable as a criterion.

Size of the Local Administrative Unit

The effectiveness of the economic index of ability is enhanced when the property assessing power is given to a single authority at the county level. More reliable data from governmental sources are available for counties than for other governmental subdivisions. There is no reason, however, why the techniques proposed in this study could not be applied to local administrative units other than

the county unit if there is a single assessing authority. The index could be first computed on a county-wide basis, and then the relative wealth of the local units within the county could be determined in proportion to their assessment rolls. Such a procedure would assume of course that uniform assessment policies prevailed within the county.

Influence of Time on a Formula

How dependable can a 1950 formula be that is calculated from 1945 or 1940 data? How often should the index be recomputed? The recognized trend of population shift from rural to urban areas would tend to change the relative position of the local administrative units. Although aggregate values of the various measures can be expected to fluctuate with the general prosperity, the relative position of the respective counties would be expected to remain relatively stable. For this reason it is not only permissible but sometimes necessary to use data for different years in developing an index of ability. A statistical lag of at least two or three years is unavoidable in the actual application of this index, because some data are not published until some time after the year of the census.

Periodic readjustment of the weights of the formula using the most current data has been suggested as a means of maintaining the precision of the index of ability. The school law of Texas[2]

[2]Texas School Law, 1949.

provided for a recomputation of the index every four years. Cornell concluded that the errors of precision between the intervals at which new statistics were reported may be disregarded.[3] Indices of relative ability have not been in actual use long enough to say how often readjustments of weights should be made. It is suggested that the weights be recalculated at least every four years until experimental evidence shows that redetermination of the weights need not be made until the elapse of a longer period of time.

Use of the Index of Relative Taxpaying Ability

There are three positions a state may take with regard to the assessment of property. If the state gives authority to a central body to make property assessments, there is no need of an index of relative taxpaying ability. However, if the state is not given the power of assessment, but only supervision of local assessments, an index is particularly useful as a check on local assessing bodies in providing equitability. The third position prevails in most of the forty eight states--the power of assessment is jealously and completely held by local assessing bodies. Under this structure the index of relative taxpaying ability provides a means of distributing equitably the financial burden of supporting an adequate statewide educational program. As the concept of equality of educational

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[3]Cornell, op. cit., p. 82.

opportunity becomes increasingly understood and accepted, and as long as the power of assessment is decentralized, it is expected that this instrument will find wider application.

APPENDIX A

Copy of Letter from Mordecai Ezekiel in Answer to Query from Writer Concerning Part Determination

Dear Mr. Lee,

....The measure you refer to, the coefficient of part correlation, is discussed on pages 182 and 183 of the first edition of my text book "Methods of Correlation Analysis," but was relegated to a footnote (page 213) of the second edition.

As stated on page 183, this coefficient measures for a linear multiple correlation equation determined by least square methods, what the correlation is between each independent variable and the variation remaining in the dependent, after the estimated (net regression) effects of the other independent variables have been eliminated. This applies, of course, to the set of independent variables used in computing the net regression coefficients involved; that is, restating your equation

$$\left[X_p = AX_1 + A_2X_2 + A_3X_3 + A_4X_4\right]$$

to my notation

$$X_1 = a_{1.2345} + b_{12.345}X_2 + b_{13.245}X_3 + b_{14.235}X_4 + b_{15.234}X_5$$

(the use of the subscripts on the net regression coefficients, "b", is important, because it defines precisely which other independent variables were taken into account in calculating the value of each "b"). If you then calculate, arithmetically, the value of

$$\sum_{345} = b_{13.245}X_3 + b_{14.235}X_4 + b_{15.234}X_5$$

for the particular values of X_3, X_4, and X_5 for each observation, and calculate

X_1 corrected $X_1 - \sum_{345}$

for each observation, and then compute the simple correlation, $\bar{r}$, of

X_1 corrected with X_2,

the correlation you will get is that given by my equations for $12\bar{3}45$.

The mathematical derivation of the formula, proving this statement, is given in the book as note 9 of the technical appendix. The square of this correlation indicates, as stated on page 183, the proportion of the variance not explained by the other independent variables included in the multiple regression equation which can be explained by each particular independent variable in turn. (See note 4 of the technical appendix for proof of this property of r^2).

I do not understand the statement in the next to the last paragraph on the first page of your letter, that you cannot use any technique which might produce negative weights for the independent variables. If the several independent variables are highly correlated or if one of them does in fact have a negative relationship with the dependent variable (such as, for example, if you take the proportion of unemployed or unemployable persons in each district as one of your independent variables) the multiple regression equation should give a negative value for the corresponding net regression coefficient. This does not disqualify the method, but only shows its accuracy. At the same time, the coefficient of part determination, being the square of the part correlation, would of course have a positive value whether the part correlation coefficient was positive or negative.

If you do adjust the total of the several coefficients to unity, you should so state in presenting them, as that is an arbitrary adjustment which modifies the meaning from the "percent explanation" given in my text.

I am interested in the fact that you have found a significant use for this measure, which I had dropped from the measures recommended for use in the second edition.

. . . . . .

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BIBLIOGRAPHY

Cornell, Francis G. A Measure of Taxpaying Ability. New York: Bureau of Publications, Teachers College, Columbia University, 1936.

Croxton, Frederick E., and Cowden, Dudley J. Applied General Statistics. New York: Prentice-Hall, Inc., 1939.

Cubberley, E. P. School Funds and Their Apportionment. Contributions to Education No. 2. New York: Bureau of Publications, Teachers College, Columbia University, 1906.

Education in North Carolina Today and Tomorrow. The Report of the State Education Commission. The United Forces for Education, Raleigh, 1948.

Ezekiel, Mordecai. Methods of Correlation Analysis. New York: John Wiley & Sons, 1930.

Florida Statutes. Committee Substitute for House Bill No. 146. A bill to be entitled "An Act Relating To Education," 1947.

The Forty Eight State School Systems: A Study of The Organization, Administration, and Financing of Public Elementary and Secondary Education. Edited by Francis S. Chase and Edgar L. Morphet. Chicago: The Council of State Governments, 1949.

Guides to the Development of State School Finance Programs. Committee on Tax Education and School Finance, National Education Association of the United States, 1949.

Hendricks, Walter A. "The Use of Relative Residuals in the Application of the Method of Least Squares." Annals of Mathematical Statistics, 2:458-78; November 1931.

Holzinger, Karl J. Statistical Methods for Students in Education. Boston: Ginn and Company, 1928.

Johns, R. L. State and Local Administration of School Transportation. New York: Bureau of Publications, Teachers College, Columbia University, 1928.

_____An Index of the Financial Ability of Local School Systems To Support Public Education. Mimeographed Report under the Auspices of the Alabama State Dept. of Education. 1938.

Lawler, Eugene S. The Amount of New Aid Required for State Equalization Programs. New York: Bureau of Publications, Teachers College, Columbia University, 1932.

McLure, William P. Let Us Pay for the Kind of Education We Need. Bureau of Educational Research, University, Mississippi, 1948.

Mort, Paul R. and Others. Federal Support for Public Education. New York: Bureau of Publications, Teachers College, Columbia University, 1936.

_____ The Measurement of Educational Need. New York: Bureau of Publications, Teachers College, Columbia University, 1924.

_____ Research Problems in School Finance. Report of the National Survey of School Finance. Washington, D. C.: The American Council on Education, 1933.

_____ State Support for Public Education. Report of the National Survey of School Finance. Washington, D. C.: The American Council on Education, 1933.

Norton, John K. The Ability of the States to Support Education. Washington: The National Education Association, 1926.

Public Schools of South Carolina. A Report of the South Carolina Education Survey Committee. Division of Surveys and Field Services, George Peabody College for Teachers, Nashville, 1948.

Richardson, C. H. An Introduction to Statistical Analysis. New York: Harcourt, Brace and Co., 1944.

Smith, B. B. "Correlation Theory and Method Applied to Agricultural Research," Bureau of Agricultural Economics. United States Department of Agriculture, 1926.

State Support of Public Schools in Tennessee. Tennessee Educational Bulletin. Nashville, 1948.

Strayer, George D., and Haig, Robert Murray. The Financing of Education in the State of New York. A Report Reviewed and Presented by The Educational Finance Inquiry Commission Under the Auspices of The American Council on Education. New York: The Macmillan Co., 1923.

A Survey of Public Education of Less Than College Grade In Georgia. A Report to the General Assembly of Georgia by its Special Committee on Education, Atlanta, 1947.

Texas School Law. Foundation School Program Act. 1949.

U. S. Bureau of the Census. Sixteenth Census of the United States: 1940. Population. Washington: Government Printing Office, 1942.

U. S. Bureau of the Census. Twelth Census of the United States: 1900. Population: Number and Distribution of Inhabitants. Washington: U. S. Government Printing Office.

Wolfenden, Hugh H. The Fundamental Principles of Mathematical Statistics. Toronto: The Macmillan Company of Canada, 1942.

BIOGRAPHICAL ITEMS

Robert Edmund Lee was born in 1919, Covington, Kentucky. He graduated cum laude from Washington & Lee University with a B.A. in Education. The Master of Arts in Mathematics was conferred by Vanderbilt University. Graduate work in Mathematics and Education was taken at the Universities of Michigan, North Carolina, and Florida. In the Navy he was Instructor of Navigation at U.S.N.R. Midshipmen's School, Northwestern University and did administrative work in the Educational Services overseas. After the war he was Instructor in Mathematics briefly at North Carolina State College and the University of North Carolina. At the University of Florida he has been an Instructor of Mathematics for two years and a teaching assistant in the College of Education for a year and a half. He is a member of Beta Theta Pi and Phi Delta Kappa.

This dissertation was prepared under the direction of the Chairman of the candidate's Supervisory Committee and has been approved by all members of the Committee. It was submitted to the Graduate Council and was approved as partial fulfillment of the requirements for the degree of Doctor of Education.

February 4, 1950

T. M. Simpson
Dean

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